ANSWER KEY Study Guide – Semester 2 Final

Directions: Answer each question as directed. Where necessary, SHOW ALL WORK and provide a thorough explanation.

1. Which postulate (SSS or SAS) can be used to prove that the triangles above are congruent? **SAS**

2. If $\triangle SJL \cong \triangle DMT$, which segment corresponds to $\overline{LS}$? Write your answer on the lines below and EXPLAIN how you found your answer.

   Correct Answer: $\overline{TG}$

   Using the congruence statements, corresponding segments are listed in the same order in the congruence statements. $\overline{LS}$ is the $3^{rd}$ and $1^{st}$ letters in the first triangle, so you must choose the $3^{rd}$ and $1^{st}$ letters in the $2^{nd}$ triangle.

   For numbers 3 and 4, write a 2-column proof OR a paragraph proof.

3. Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$, $\angle R \cong \angle E$, $\angle T \cong \angle F$

   Prove: $\triangle PRT \cong \triangle DEF$

   $\triangle PRT \cong \triangle DEF$ by AAS. All of the necessary information was given, two sets of congruent angles and a non-included side. $\angle R \cong \angle E$ and $\angle T \cong \angle F$ were given, which satisfies the two sets of congruent angles. $\overline{PR} \cong \overline{DE}$ OR $\overline{PT} \cong \overline{DF}$ can be used as the non-included side.
4. **Given:** \( \overline{AB} \cong \overline{CB} \), \( \overline{AD} \cong \overline{CD} \), \( \overline{BD} \cong \overline{BD} \)

**Prove:** \( \angle ABD \cong \angle CBD \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. ( \overline{AB} \cong \overline{CB} ), ( \overline{AD} \cong \overline{CD} ), ( \overline{BD} \cong \overline{BD} )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \triangle ABD \cong \triangle CBD )</td>
<td>SSS</td>
</tr>
<tr>
<td>3. ( \angle ABD \cong \angle CBD )</td>
<td>CPCTC</td>
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- **Hint:** Remember, you always prove sides or angles congruent in triangles with CPCTC – Corresponding Parts of Congruent Triangles are Congruent.
- **Hint:** SAS, SSS, ASA, and AAS are only used to prove TRIANGLES congruent.

**Directions:** Write a 2-column proof for 5 and 6. **Pay special attention to what you are being asked to prove (e.g. triangles? sides? angles?).**

5. **(Use ASA) – two angles and the included side**

**Given:** \( S \) is the midpoint of \( QR \).
\( \overline{QR} \parallel \overline{TU} \)

**Prove:** \( \triangle QSR \cong \triangle TSU \)

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<tr>
<td>1. ( S ) is the midpoint of ( QT ). ( \overline{QR} \parallel \overline{TU} )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \overline{QS} \cong \overline{TS} )</td>
<td>Definition of Midpoint</td>
</tr>
<tr>
<td>3. ( \angle Q \cong \angle T )</td>
<td>Alternate Interior Angles formed by a transversal cutting two parallel lines are congruent.</td>
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<tr>
<td>4. ( \angle RSQ \cong \angle UST )</td>
<td>Vertical Angles are congruent</td>
</tr>
<tr>
<td>5. ( \triangle QSR \cong \triangle TSU )</td>
<td>ASA</td>
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</table>
6. (Use AAS)

Given: \( \angle D \cong \angle F \)
\[ \overline{GE} \text{ bisects } \angle DEF. \]

Prove: \( \overline{DG} \cong \overline{FG} \)

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| 1. \( \angle D \cong \angle F \)  
\[ \overline{GE} \text{ bisects } \angle DEF. \] | Given |
| 2. \( \angle DEG \cong \angle FEG \) | Definition of Angle Bisector |
| 3. \( \overline{GE} \cong \overline{GE} \) | Reflexive Property |
| 4. \( \triangle DEG \cong \triangle FEG \) | AAS |
| 5. \( \overline{DG} \cong \overline{FG} \) | CPCTC |

Hint: Since we have to prove segments congruent, you FIRST must prove the triangles are congruent, THEN show that the segments are congruent by CPCTC.

Use the picture and the information below to answer questions 7 – 8.

An architect used the window design in the diagram to the left when remodeling an art studio. \( \overline{AB} \) and \( \overline{CB} \) each measure 3 ft.

7. Using 2 – 3 sentences, justify why \( \angle A \cong \angle C \).

Since \( \overline{AB} \) and \( \overline{CB} \) each measure 3 feet, \( \triangle ABC \) is an isosceles triangle. Therefore, the angles opposite \( \overline{AB} \) and \( \overline{CB} \) are also congruent, which are \( \angle A \) and \( \angle C \).
8. Considering SSS, AAS, ASA, or SAS, given that \( \angle A \cong \angle C \), can you justify why \( \triangle ABD \cong \triangle CBD \)? Why or why not?

This proof or justification cannot be made since there is not enough information given. Yes, you have two sides of the triangles being congruent, and one angle in each triangle congruent; however, in order to use SSS, you need more information. You cannot assume segment BD bisects segment AC. To use SAS, AAS, or ASA, you need to prove that the other two angles in the triangles are congruent. There is not enough information to do this.

9. What is the value of \( x \) in the figure below?

Since the triangle is isosceles, \( \angle A \) and \( \angle C \) are congruent. Thus, \( 6x + 6 + 6x + 6 + 2x = 180 \), using the Triangle Angle Sum Theorem.

Answer: \( x = 12 \)

10. Solve the proportion below for \( x \).

\[
\frac{12}{x} = \frac{21}{84}
\]

Answer: \( x = 48 \)

11. A magician has 40 marbles in a bag for a magic trick. There are 9 red marbles, 10 white marbles, 11 orange marbles, 5 blue marbles, and 5 black marbles. What is the ratio of orange marbles to red marbles?

Answer: 11 to 9
12. The ratio of the measures of the sides of a triangle is 7:9:12, and its perimeter is 84 inches. Find the measure of each side of the triangle.

\[7x + 9x + 12x = 84; \quad x = 3\]

**Answer:** 21 inches, 27 inches, 36 inches

13. The ratio of the measures of the sides of a triangle is 6:7:9, and its perimeter is 77 centimeters. Find the measure of each side of the triangle.

\[6x + 7x + 9x = 77; \quad x = 3.5\]

**Answer:** 21 cm, 24.5 cm, 31.5 cm

14. A worker at an automobile assembly plant checks new cars for defects. Of the first 280 cars he checks, 4 have defects. If 10,500 cars will be checked this month, predict the total number of cars that will have defects.

Set up and solve the following proportion: \[\frac{280}{4} = \frac{10,500}{x}\]

**Answer:** x = 150 cars
Use the figures below to answer questions 15 and 16 given that \( \triangle RST \sim \triangle MNP \).

![Diagram](image)

15. What is the value of \( y \) in \( \triangle MNP \)?

Set up a proportion and solve; since the two triangles are similar, their corresponding sides are proportional.

\[
\frac{32}{16} = \frac{38}{y}
\]

Answer: \( y = 19 \)

16. What is the value of \( x \) in \( \triangle RST \)?

Set up a proportion and solve; since the two triangles are similar, their corresponding sides are proportional.

\[
\frac{32}{16} = \frac{x}{13}
\]

Answer: \( x = 26 \)
17. If \( AB \sim PQ \), find the scale factor of \( AB \) to \( PQ \).

\[
\text{Scale factor: } \frac{20}{25} = 0.8 \text{ (to go from big polygon to smaller one)} \quad \text{OR} \quad \frac{25}{20} = 1.25 \text{ (to go from small polygon to big polygon)}
\]

18. The figures above are similar. What is the value of \( x \)?

Since the two triangles above are similar, we can set up the proportion \( \frac{x + 15}{18} = \frac{40}{30} \). Cross multiplying yields \( 720 = 30(x + 15) \). Using the distributive property on the right side of the equations yields \( 720 = 30x + 450 \). Solving this equation yields \( x = 9 \).

19. Compare and contrast similar polygons with congruent polygons.

Congruent polygons have all corresponding parts that are congruent; precisely, all corresponding angles are congruent and all corresponding sides are congruent. Similar polygons are comparable to congruent polygons in that all corresponding angles are congruent. However, similar polygons are different from congruent polygons in that they DO NOT have congruent corresponding sides. In similar polygons, corresponding sides are proportional.

20. The bottom of a ladder must be placed 3 ft. from a wall. The ladder is 12 feet long. How far above the ground does the ladder touch the wall?

\( \sqrt{135} \approx 11.61 \text{ feet} \); use the Pythagorean Theorem with 12 as the hypotenuse (value of c) and 3 as one of the values of the legs.
21. A soccer field is a rectangle 90 meters wide and 120 meters long. The coach asks players to run from one corner to the other corner diagonally across. What is this distance?

The distance players must run is the hypotenuse of a right triangle. 90 and 120 are the lengths of the legs. Use the Pythagorean Theorem to get 150 meters.

22. The figure to the left is a parallelogram. What is the value of x and y?

The diagonals of a parallelogram bisect each other. Thus, 2y = 28, so y = 14. Because of the two hash marks on one half of both diagonals, both diagonals are congruent to each other. Thus, 2y = 4x = 28. Using 4x = 28, x = 7.

23. The figure to the left is a rhombus. What is the value of x and y?

x = 15, y = 7.5

The diagonals of a rhombus are perpendicular bisectors of each other which means four right triangles are formed inside of the rhombus. To find the value of x, we can use the Triangle Angle Sum Theorem (2x + 60 + 90 = 180) that yields x = 15. To calculate the value of y, we know that 2x = 4y because the opposite sides of a parallelogram are parallel, and thus 2x and 4y are the measurements of a pair of congruent alternate interior angles. Plugging in x into 2x yields 30. Thus, 4y = 30 which yields y = 7.5.
24.

Given that \( \overline{HJ} \parallel \overline{KL} \), using a paragraph format, construct a valid argument as to why \( \triangle GLK \sim \triangle GJH \).

To prove that \( \triangle GLK \sim \triangle GJH \), we have to show that corresponding sides are proportional to each other OR corresponding angles are congruent to each other. Since no side lengths are given, we can show that two sets of corresponding angles are congruent to each other. \( \angle G \) is in both triangles, so \( \angle G \cong \angle G \) by the Reflexive Property. \( \angle JHK \cong \angle LKG \) since they are corresponding angles formed by a transversal \( \overline{HG} \) cutting two parallel lines \( \overline{HJ} \) and \( \overline{KL} \). We can show that \( \angle HJL \cong \angle KLG \) by the Third Angle Theorem OR use the fact that they are corresponding angles formed by a transversal \( \overline{LG} \) cutting two parallel lines \( \overline{HJ} \) and \( \overline{KL} \).

Since all three pairs of corresponding angles are congruent, then the two triangles are similar \( \triangle GLK \sim \triangle GJH \).
25. a. What type of angles are 6 and 5? Linear pair
   b. What type of angles are 7 and 2? Alternate Exterior
   c. What type of angles are 5 and 4? Alternate Interior
   d. What type of angles are 6 and 4? Consecutive Interior

26. Form sin J, cos J, and tan J using the figure below. (SOHCAHTOA)

   \[
   \sin J = \frac{12}{20} = 0.6 \quad \cos J = \frac{16}{20} = 0.8 \quad \tan J = \frac{12}{16} = 0.75
   \]